REFLECTIONS ON THE FATE OF SPACETIME

String theory carries the seeds of a basic change in our ideas about spacetime and in other fundamental notions of physics.

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Our basic ideas about physics went through several upheavals early this century. Quantum mechanics taught us that the classical notions of the position and velocity of a particle were only approximations of the truth. With general relativity, spacetime became a dynamical variable, curving in response to mass and energy. Contemporary developments in theoretical physics suggest that another revolution may be in progress, through which a new source of "fuzziness" may enter physics, and spacetime itself may be reinterpreted as an approximate, derived concept. (See figure 1.) In this article I survey some of these developments.

Let us begin our excursion by reviewing a few facts about ordinary quantum field theory. Much of what we know about field theory comes from perturbation theory; perturbation theory can be described by means of Feynman diagrams, or graphs, which are used to calculate scattering amplitudes. Textbooks give efficient algorithms for evaluating the amplitude derived from a diagram. But let us think about a Feynman diagram intuitively, as Feynman did, as representing a history of a spacetime process in which particles interact by the branching and rejoining of their world-lines. For instance, figure 2 shows two incident particles, coming in at a and b, and two outgoing particles, at c and d. These particles branch and rejoin at spacetime events labeled x, y, z and w in the figure.

According to Feynman, to calculate a scattering amplitude, one sums over all possible arrangements of particles branching and rejoining. Moreover, for a particle traveling between two spacetime events x and y, one must in quantum mechanics allow for all possible classical trajectories, as in figure 3. To evaluate the propagator of a particle from x to y, one integrates over all possible paths between x and y, using a weight factor derived from the classical action for the path.

So when one sees a Feynman diagram such as that of figure 2, one should contemplate a sum over all physical processes that the diagram could describe. One must integrate over all spacetime events at which interactions—branching and rejoining of particles—could have occurred, and integrate over the trajectories followed by the particles between the various vertices. And, of course, to actually predict the outcome of an experiment, one must (as in figure 4) sum over all possible Feynman diagrams—that is, all possible sequences of interactions by which a given initial state can evolve into a given final state.

This beautiful recipe—formulated in the early days of quantum field theory—brought marvelous success and efficient, precise computations. Yet this recipe also exhibits certain of the present-day troubles in physics. One important property of a Feynman graph is that the graph itself, regarded as a one-dimensional manifold, is singular; that is, at the branching and joining points, the graph does not look like a true one-dimensional manifold. Everyone can agree, in figure 2 for instance, that x, y, z and w were the spacetime events at which interactions occurred. Two central difficulties spring directly from this:

Infinities. Quantum field theory is plagued with infinities, starting with the infinite electrostatic self-energy of the electron. The infinities come from the singularities of the Feynman diagrams. For instance, in figure 2, the potential infinities come from the part of the integration region where the spacetime events x, y, z and w all nearly coincide. Sometimes the infinities can be "renormalized" away; that is the case for electrodynamics and for the weak and strong interactions in the Standard Model of elementary-particle physics. But for gravity, renormalization theory fails, because of the nature of the inherent nonlinearities in general relativity. So we come to a key puzzle: The existence of gravity clashes with our description of the rest of physics by quantum fields.

Too Many Theories. There are many quantum field theories, depending on many free parameters, because one can introduce fairly arbitrary rules governing the branching and joining of particles. For instance, one could permit higher-order branchings of particles, as in figure 5. With every elementary branching process, one can (with certain restrictions) associate a "coupling constant," an extra factor included in the evaluation of a Feynman diagram. In practice, the Standard Model describes the equations that underlie almost all the phenomena we know, in a framework that is compelling and highly predictive—but that also has (depending on precisely how one counts) roughly seventeen free parameters whose values are not understood theoretically. The seventeen parameters enter as special factors associated with the singularities of the Feynman diagrams. There must be some way to reduce this ambiguity!

String theory

We have one real candidate for changing the rules; this is string theory. In string theory the one-dimensional trajectory of a particle in spacetime is replaced by a two-dimensional orbit of a string. (See figure 6.) Such strings can be of any size, but under ordinary circum-

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